Caribbean Educational Research Journal
Volume 7, Issue 2, December 2022, 98-125

MATHEMATICS COACHES: THEIR PREPAREDNESS TO TEACH MATHEMATICS USING A PROBLEM-SOLVING APPROACH

Avalloy McCarthy-Curvin*
The University of the West Indies, Mona

Lois George
The University of the West Indies, Mona

Camella Buddo
The University of the West Indies, Mona

Abstract: *The New Standards-Based Mathematics Curriculum (NSC) of the Jamaican education system, like many mathematics curricula in other countries, has as one of its core objectives to engage learners in Science, Technology, Engineering and Mathematics (STEM) education. Problem solving is a key component of these STEM areas. Through its annual staging of the Grade 9 Mathematics Problem-solving Competition, the School of Education at The University of the West Indies (UWI), Jamaica interfaces with secondary school students and their mathematics coaches. In assessing students' solutions to the mathematics problems posed during the competition, the level of preparation of participants for the competition became a concern. This research therefore investigated mathematics coaches' descriptions of a problem-solving approach to teaching and learning mathematics and perceptions of preparedness to use a problem-solving approach to teach mathematics. It further explored how mathematics coaches guide students through the problem-solving process and what problem-solving strategies are taught by these coaches. A survey research design involving 31 mathematics coaches was used for the research. Both quantitative and qualitative data were collected using a questionnaire. The findings revealed that although the mathematics coaches (who are classroom teachers) feel confident that they are able to teach mathematics using a problem-solving approach, they do not appear to have the requisite knowledge and skills to effectively teach mathematics using a problem-solving approach. The findings suggest that there is a need for trainers of teachers to evaluate and strengthen their programmes on aspects of problem solving for mathematics teaching and learning.*

Keywords: *mathematics, problem solving, mathematics coaches, teaching, STEM*

* Corresponding Author (avalloymccarthy6@yahoo.com)
ISSN 1727-5512

INTRODUCTION

Research Background

A fundamental aim of the mathematics curriculum is to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help solve everyday problems (Jamaica Ministry of Education, 2013). However, at the end of secondary schooling in Jamaica, many graduates are still unable to apply outside of the context of the classroom the mathematics knowledge they have learnt and to problem solve (Jamaica Ministry of Education, 2013). This problem is not unique to Jamaica. as countries such as the United States, Turkey, United Kingdom, Nigeria, Botswana and some other Caribbean countries (Darling-Hammond, 2010; Mapolelo & Akinsola, 2015) struggle to produce graduates who are innovators and problem solvers. This has several implications for a country's development in moving towards a Science, Technology, Engineering and Mathematics (STEM) related workforce.

STEM workers play a key role in the sustained growth and stability of many countries' economies. Mathematics knowledge is required in all these STEM areas. However, the mathematics knowledge that is required is the ability to use the knowledge and tools learnt in the mathematics classroom to solve problems that will enrich the lives of mankind. In other words, what is required to grow an economy and create innovations are citizens who can observe what is going on in their environments and identify how to create change or improvement. In Singapore, teachers employ a problem-solving approach to the teaching and learning of mathematics, and that country has one of the most developed economies in the world (Darling-Hammond, 2010). The ability to solve problems becomes pertinent for economic growth and development.

It is plausible to argue that if we want to create a country where citizens are problem solvers, then the education system must cater to the development of mathematical problem solving. In Jamaica, problem solving is mainly emphasized in the National Mathematics Policy and in the curriculum. Therefore, it becomes necessary for teachers of mathematics to have the knowledge and skills required to develop students' problem-solving skills.

Impetus for the Research

Since 2001/2002, the Mathematics Unit in the School of Education at The University of the West Indies, Mona Campus has been hosting an annual Grade 9 Mathematics Problem-solving Competition. This competition emerged out of a course titled *Problem-solving and*

Investigations that is offered in the undergraduate Mathematics Education programme. For an assignment, course participants were required to carry out a mini-project in a class at a secondary school; to use problem solving as a teaching approach and to facilitate the development of problem-solving skills in students. This project has "blossomed" into the competition among Grade 9 students across secondary schools in Jamaica. This competition engages the participants in reasoning, as well as critical and creative thinking, as they work cooperatively in their schools' four-member teams to solve mathematics problems.

All secondary schools across the fourteen parishes of Jamaica are eligible to enter the competition. Schools are required to enter four students from their Grade 9 cohort. The expectation is that specific mathematics teachers would prepare the students for the competition by employing a problem-solving approach to the teaching and learning of mathematics. In this article, the mathematics teachers who are responsible for preparing the students for the competition are referred to as mathematics coaches. These mathematics coaches' own perspectives of problem solving determine the approaches they use in the classrooms for teaching mathematics and for preparing their students for the Grade 9 Mathematics Problem-solving Competition.

Schools are aware of the date of the competition at least three months in advance. This provides time for the mathematics coaches to prepare their Grade 9 students for the competition. It is also noteworthy to highlight that some schools have on-going preparation for the competition.

Statement of the Problem

Over the time that the Grade 9 Mathematics Problem-solving Competition has been held, the poor problem-solving abilities of some of the Grade 9 students who entered the competition have raised concerns about their mathematics coaches' ability to enable the development of students' problem-solving skills. In other words, as organizers of the competition, we questioned whether the mathematics coaches have the knowledge and skills necessary to effectively enable students to develop problem-solving skills.

Rationale and Positionality

Hattie (2003) reports that outside of student factors such as ability and interest, the teacher factor has the greatest impact on students' learning gains. This suggests that one way to improve students' learning outcomes is to improve the variable (teachers) that has the greatest external impact on student outcomes. In the context of the Grade 9 Mathematics

Problem-solving Competition, we believe that if we (teacher educators in the School of Education) can identify where there might be shortfalls in mathematics coaches' ability to enable the development of students' problem-solving skills, then we would be able to provide the appropriate training to coaches in order to remedy the problem areas. Our hope is to ultimately improve students' problem-solving skills. It is against this backdrop that we reached the conclusion to investigate the mathematics coaches' understanding of teaching and learning mathematics using a problem-solving approach, as well as their preparedness for teaching mathematics using a problem-solving approach. This research explores the following questions:

1. How do mathematics coaches describe a "problem-solving approach to teaching and learning mathematics"?
2. What are mathematics coaches' perceptions of their level of preparedness to use a problem-solving approach to teach mathematics?
3. How do mathematics coaches guide students through the problem-solving process?
4. What problem-solving strategies are taught by the mathematics coaches?

Significance of the Research

Consistent with international positions (e.g., National Council of Teachers of Mathematics, 2014), the Government of Jamaica, through the National Mathematics Policy Guidelines put forward by Ministry of Education (2013), has asserted that students' development of problem-solving skills should have a central role in the teaching and learning of mathematics at all educational levels. Although the use of problem solving is a requirement of the Ministry of Education (MoE) as per the curriculum in Jamaica, there are no research studies which have examined the aspects of problem-solving that are explored in this research. In fact, teachers' preparedness to teach mathematics using a problem-solving approach has been sparsely researched. The current research therefore aims to close a gap in the literature by providing insights into some of the issues surrounding mathematics teachers' ability to employ a problem-solving approach to the teaching and learning of mathematics.

In addition, as teacher educators of The UWI, there is an expectation that we seek to fulfill key roles such as providing policy implementation guidance to the MoE and assisting teachers to integrate problem solving effectively into their mathematics lessons. To achieve this, it is important to understand what views of problem solving these teachers hold and what strategies for implementation they are currently using. This will serve as a springboard for creating programmes to strengthen teachers' use of the problem-solving approach in their mathematics classes, as well as to promote and support the MoE's mandate relating to deeply integrating problem solving within mathematics teaching and learning.

LITERATURE REVIEW

Defining a Problem

Within the mathematics education literature, there are varied conceptualizations of mathematical problems and problem solving (Liljedahl et al., 2016; Woodward et al., 2012). One key feature that is frequently linked with mathematical problem solving is novelty (Cai & Lester, 2010; Schoenfeld, 2013). This suggests that the problem solver does not immediately know what to do to get a solution. This feature of novelty highlights the non-routine character of problem solving and sets mathematical problems apart from exercises and routine problems (Beghetto, 2017; National Council of Teachers of Mathematics, 2014). Generally, exercises and routine problems are "familiar problems that, although not eliciting an automatic memorized answer, can be solved by applying a well-known procedure. Although the problem solver does not immediately know the answer to a routine problem, the problem solver *does* know how to arrive at an answer" (Mayer, 2012, p. 40). Mayer further states that, conversely, non-routine problems are unfamiliar problems (to the problem solver) for which the individual solving the problem does not have a previously-formulated or known method of solving and must therefore use productive thinking to generate a creative or novel solving procedure. Higher order thinking skills, such as reasoning and critical analysis, must be utilized by individuals engaged in solving non-routine problems and in mathematical problem solving (National Council of Teachers of Mathematics, 2014; Woodward et al., 2012). On the other hand, reproductive thinking, whereby a previously-known procedure is applied to solving a problem is used when individuals engage with routine problems. When the term "problem" is used in this article, the focus is on non-routine mathematical problems.

Mathematical problems have also been categorized as well-defined or ill-defined. The former type of problem "has a clearly specified given state, goal state and operators that may be applied to solving the problem" whilst the latter "lacks a clear specification of one or more of the given states, the goal state or the operators" (Mayer, 2012, p. 40). An example of a well-defined problem for a middle attaining Grade 11 student is: Solve for x; $\frac{1}{2}x + 3 = \frac{3}{5}x$, since they can indicate the given state (i.e. the equation), the goal state (i.e. x = __) and the operators or procedure(s) for solving. The assignment requiring students to write an essay on mathematical problem solving is an ill-defined problem since the various states cannot be readily specified.

THEORETICAL UNDERPINNING: THE GEORGE POLYA MODEL

Although researchers and practitioners within the domain of mathematics education have formulated several models to describe general problem solving (for example: Dewey, 1933; Lerman, 1983; Mason et al., 2010; Polya, 1945; Schoenfeld, 1985), George Polya, through his seminal book *How to solve it,* is recognized as the main individual who initiated investigations for helping students to engage in problem solving in mathematics (English & Sriraman, 2010). Zollman (2010, p. 297) also notes that "the teaching of strategies to problem solve in mathematics {is} rooted in Polya's heuristics". These are detailed in the next section. Some mathematics educators and researchers consider Polya's heuristics to be one of the most widely used set of strategies in the teaching and learning of mathematics problem solving (Fan & Zhu, 2007). Polya's perspective has been used successfully globally, for example, in jurisdictions such as Singapore and the United States (Fan & Zhu, 2007; Toh et al., 2011), Pakistan (Perveen, 2010) and China (Leong et al., 2010).

While the work of George Polya as it relates to problem solving has been well-received within the mathematics education community and initially influenced the model of problem solving applied in schools (Schoenfeld, 1987), there have been several criticisms of this model. First, research has consistently found that the teaching of heuristics and problem-solving strategies by classroom teachers has not led to improving students' problem-solving abilities (English & Sriraman, 2010; Lesh & Zawojewski, 2007; Liljedahl et al., 2016; Schoenfeld, 1987). Instead, Cai and Lester (2010) argue that it is when students engage in problem solving that their problem-solving capabilities improve and not when they are taught about strategies for engaging with mathematical problems. Another critique put forward by Schoenfeld (1987, p. 288) was that "the heuristic strategies did not contain enough detail for people unfamiliar with the strategies to be able to implement them. Polya's characterizations were labels under which families of related strategies were subsumed" and so were too general to be useful to students.

Despite the criticisms of Polya's well-established model of problem solving, we have adopted it as the theoretical underpinning for this research. This is because, in the Jamaican educational context, the Ministry of Education (MoE) promotes it as one of the main models for engaging students in problem solving. Furthermore, to date, many mathematics workshops, seminars and initiatives for teachers have been conducted by the Ministry of Education (MoE) and so we surmised that this would be the model with which teachers were most familiar and are most likely to use as they prepared students for the Grade 9 Mathematics Problem-solving Competition.

Teachers' Understanding of Problem Solving

Research on teachers' understanding of problem-solving shows that their levels of understanding of problem solving and problem-solving pedagogy vary (Cai & Lester, 2010). Some mathematics teachers perceive problem solving as the application of mathematics to real life; others associate it with non-routine tasks, while another set of teachers associate it with worded questions (Lester & Kehle, 2003; Stein et al., 2003).

The fact that teachers hold different understandings of problem solving has influenced differences in their approach to teaching students to develop problem-solving skills (Lerman, 1983; National Council of Teachers of Mathematics, 2014). For example, some teachers view effective teaching of mathematics using a problem-solving approach as providing students with a problem and then standing back and letting them struggle until they find a solution, while other teachers would scaffold students through the problem-solving process (National Council of Teachers of Mathematics, 2014).

How Teachers Incorporate Problem Solving into Mathematics Teaching

There are vast differences in how teachers across the globe incorporate problem solving in their mathematics classrooms and lessons (Lester, 2013). These are the result of varied factors such as exposure to training, school culture, teacher self-efficacy, beliefs, assumptions and conceptualizations of the nature of mathematics learning, teaching, instructional goals and problem-solving in general, as well as the role of problem solving in mathematical learning in particular (Lester, 2013; Lester & Cai, 2015). In early empirical work, Lerman (1983) found that teachers who regard mathematics as problem-centred viewed problem solving as a way to teach mathematics. In other words, they adopted a teaching approach that viewed problem solving as a means through which mathematical concepts, processes and procedures are learnt (Lester & Cai, 2015; Van de Walle et al., 2010).

Lerman also discovered that teachers who regarded mathematics as a compilation of procedures and facts deemed problem solving to be the concluding activity after learning some mathematical concept, fact and/or procedures. Using this approach, the teacher is teaching *for* problem solving and considers problem solving to be the *end* result of teaching.

For example, after teaching the concepts and skills associated with the topic of hire purchase a teacher may present students with problems on the topic to solve.

A third way of incorporating a problem-solving approach into mathematics teaching was via a collection of heuristics (Polya, 1945). The heuristics put forward by Polya (1945) included: understanding the problem, planning, carrying out the plan, and looking back.

The first of the four heuristics speaks to having learners understand the problem by "extracting and assimilating information from the givens, determining the goal of the problem, reconstructing the problem if necessary, and introducing suitable notations whenever possible for easy reference and manipulations" (Fan & Zhu, 2007, p. 65). Teachers are required to help students understand the problem by asking them questions that will clarify their understanding of the problem. This might include asking students questions about the vocabulary that is used in the problem, asking them to restate the problem in their own words, or to identify what is required to solve the problem.

The second heuristic involves having learners devise a plan. At this stage, a learner needs to decide on a plan that they think might provide the solution and choose appropriate methods and/or approaches that may assist in this regard. The third heuristic requires learners to carry out the plan formulated in the previous step. Polya (1945) explained that at this stage, the students are required to put the plan into action and persist with the plan. He further explained that if much time is spent on the plan and it continues not to work, learners should be encouraged to discard the initial plan and choose another. The final heuristic requires learners to look back at the solution and the problem-solving process. This is undertaken to check whether the solutions are accurate, approaches are appropriate, and to reflect on the main ideas and procedures of the solutions which would allow the problem solvers to determine whether what was done seems plausible, and to generalize or extend the procedures or answers, if necessary.

Problem-solving strategies

In the literature, the most commonly referenced problem-solving strategies include: *solving a simpler equivalent problem; intelligent guessing and checking; simulating; working backwards; looking for a pattern; drawing a picture or diagram; using manipulatives; trial-and-error; trying special values or cases (specialization); listing all possibilities; acting it out; logical reasoning; making a systematic list; making suppositions; gathering and organizing data in tables; creating and using mathematical modelling* (Polya, 1945; Mason et al., 2010; Posamentier & Krulik, 2009; Woodward et al., 2012). In using a problem-solving approach to the teaching of mathematics, teachers need to make decisions regarding how best to expose students to these strategies and/or to incorporate them into their lessons. Yazgan (2016) suggests that teachers should facilitate the learning of these strategies by carefully choosing non-routine problems that promote the use of varied strategies by students. For example, the following problem would lend itself to students using strategies such as making a systematic list, drawing a picture or diagram, using manipulatives (i.e., paper strip), trial-and-error and looking for a pattern strategy.

Paper Strip Problem: Imagine a long thin strip of paper stretched out in front of you, left to right. You take the ends in your hands and place the right-hand end on top of the left. You now press the strip flat so that it is folded in half and has a crease. You repeat the whole operation on the new strip two more times. How many creases are there? How many creases will there be if the operation is repeated 10 times in total?

Teachers must also regularly use problems as part of mathematics instruction so that students become proficient in the use of a variety of strategies (Cai & Lester, 2010; Lester, 2013; National Council of Teachers of Mathematics, 2014). In addition, Woodward et al. (2012) urged teachers to expose students to multiple strategies for solving a problem through the use of demonstration, discussion and/or comparison of student work.

Teachers must be clear as to the mathematical content that underpins or is embedded within the selected problem and that the problem aligns with the lesson objectives. In keeping with this approach, "teachers must consider their unit goals and their students' background and interests when preparing problem-solving lessons" (Woodward et al., 2012, p. 8).

The National Council of Teachers of Mathematics (2014) asserts that while choosing problems that encourage problem solving is an important initial step, teachers need to facilitate student problem solving in several ways for it to be optimally effective.

> Teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge. (p. 22)

Although the Council recommends that students engage in productive struggle, teachers should not just abandon students to working on their own for extended periods, especially in early engagements. Instead, they should provide appropriate support through asking the right questions and giving suggestions that would allow students to advance in solving a given problem.

Teachers' Preparedness to Teach Mathematics Using a Problem-Solving Approach

Empirical research consistently shows that teaching quality makes a difference in students' achievement (Blömeke et al., 2016; Hattie, 2003; Telese, 2012). Based on this premise, it becomes necessary for teachers of mathematics to be adequately prepared to teach using a problem-solving approach. Teachers often report that they lack the knowledge and skills associated with incorporating problem solving meaningfully into the mathematics curriculum (Buschman, 2004; Cai & Lester, 2010). To compound this challenge, there is very little empirical research to guide teachers as to what should be done in the mathematics class to advance students' problem-solving competence (Lester & Cai, 2015). Additionally, few studies have investigated teachers' preparedness to teach using a problem-solving approach. The literature on problem solving often focused on teachers' methods of incorporating problem solving in the classroom and challenges that teachers faced in using a problem-solving approach in the classroom (Lester, 2013; Lester & Cai, 2015).

METHODOLOGY AND RESEARCH DEGISN

Research Paradigm

We adopted a pragmatic research paradigm for this research. Our aim is not focused on whether truth is objective (fixed) or created in the minds of individuals. Instead, our focus is placed on identifying "what works" to bring change and improvement to the lives of mankind. For this reason, we operated from a place where we sought to employ whatever research methodologies that provided insights into the problem we were exploring. In the case of this research, we endeavoured to understand the problem of Grade 9 students' poor problem-solving abilities and as a result we sought the most practical approach that would provide the insights we needed to address the problem.

Many of the Grade 9 students did not use appropriate problem-solving processes or strategies to help them find solutions to the problems that were assigned. We therefore surmised that it was unlikely that they would have been effectively taught these skills and chose not to utilize them when required. Therefore, we figured that the mathematics coaches (based on their roles in preparing the students for the competition) would be able to provide us with the greatest insights into the problem we sought to address, by speaking to their own understandings of problem solving and their approaches to developing problem-solving skills in their students. Additionally, our analysis of the logics that were involved in garnering data that would capture the perspectives of all our mathematics coaches led us to adopt a survey research design for this research.

Research Design

A survey research design allows researchers to respond to descriptive questions (Creswell & Creswell, 2018), such as those investigated in this research. The use of a survey was also appropriate because it allowed us to ask targeted questions, both quantitative and qualitative, in keeping with the research focus, to a potentially large number of respondents (Creswell & Creswell, 2018) in a relatively short period of time.

The Sample

Data was collected from 31 mathematics coaches who attended the 2019 Grade 9 Mathematics Problem-solving Competition with their students. A non-probability convenience sampling technique was therefore used for this research. Schools from five of the 14 parishes (numbering 31 schools) across Jamaica were represented at the competition. Therefore, the mathematics coaches who took part in the research were from five of the fourteen parishes in Jamaica. There were 14 female and 17 male mathematics coaches with a range of teaching experience from 2 to 20 years. Twenty-six of the mathematics coaches held a Bachelor of Education in Mathematics; three had a Bachelor of Science in Mathematics; one had a Bachelor of Science in Food Chemistry, and one had a Bachelor of Science in Accounting.

While across Jamaica there are 170 secondary schools, from our experience in organizing the Grade 9 Mathematics Problem-solving Competition annually, typically only approximately a quarter of the total number of secondary schools attend the competition. In 2019, the total number of mathematics coaches was 31, which is consistent with previous years. It is possible that this limited number of coaches (18.2%) may have been due to competing school commitments, lack of funding to travel to and participate in the competition, lack of confidence in students' abilities to successfully engage in solving the problems presented at the competition and/or inadequate advertising of the activity. One limitation of the small sample of schools who participated in this study is that the research findings are not generalizable to the population. Key insights, however, can be gained in relation to mathematics teachers' perceptions and enactment of a problem-solving approach in the teaching of mathematics.

The Research Instrument

A questionnaire of 10 open-ended items and 12 closed-ended items was designed to collect the data in this research (see Appendix A). The respondents were required to answer questions on a Likert scale as well as to provide short responses. The items were clustered into four headings: Demographics (This heading contains five items); Preparedness to Teach Mathematics Using a Problem-Solving Approach (This heading contains 10 items); Problem-Solving Approach to Teaching (This heading contains six items) and Challenges with Teaching Using a Problem-Solving Approach (This heading contains one open-ended item). One member of the research team constructed the questionnaire. The items were generated from the literature. For example, we queried "How do you guide students through the problem-solving process?" This was influenced by the literature reviewed, such as Polya (1945). Additionally, the heading was guided by the research questions we sought to answer. For example, to answer the research question "What are mathematics coaches' perceptions of their level of preparedness to use a problem-solving approach to teach mathematics", an item on the Likert scale was "you are knowledgeable about the problem-solving process". The respondents had the option to select from "Strongly Agree" through to "Strongly Disagree".

Reliability and Validity of the Instrument.

Although the questionnaire was designed by one member of the research team, it was reviewed by the other two members. The reviewers evaluated the items to determine whether they measured what was intended, as well as whether the items were free from ambiguity and clearly articulated. Subsequently, the questionnaire was revised based on the comments from the reviewers.

The research team consisted of lecturers from The University of the West Indies (Mona Campus). Although in hindsight, it could have been potentially beneficial to seek the input of others, such as Ministry of Education officials, in constructing the questionnaire, we did not do so. Instead, we relied on our combined experience of over twenty-five years as lecturers in mathematics education, and on the research literature to capture the key aspects related to the research foci: Teachers' conceptualization of problem-solving approaches to teaching; Preparedness to teach mathematics using a problem-solving approach and Challenges with teaching using a problem-solving approach.

The instrument was piloted to ensure its reliability. The questionnaire was administered to 21 secondary school mathematics teachers. None of the teachers who participated in the pilot survey participated in the actual study. The responses from the closed ended items were uploaded to SPSS version 22.0 where a Cronbach Alpha coefficient of .844 was found. This Cronbach Alpha coefficient was within the acceptable range of .7 to .9 (Creswell & Creswell, 2018).

It is noteworthy to highlight that although several existing instruments that measure students' ability to problem solve and teachers' ability to facilitate a problem-solving approach such as Hill (1997) and Ngang et al. (2013) were examined, none of these instruments could be used to achieve the purpose of the research, and ultimately answer the desired research questions. In other words, the items on these instruments did not align with what we sought to investigate.

DATA ANALYSIS

To answer the research questions, the data was analysed using several different strategies. For Research Question 1, the description of a problem-solving approach that was provided by the mathematics coaches was examined and each description was assigned one of three codes based on the subject of the description. The codes that were assigned were: feature of novelty; real life questions and difficult tasks. Following this, the number of responses under each of the codes was tallied and the percentage calculated. To answer Research Question 2, the closed ended data was analysed using SPSS version 22.0, where descriptive statistical analysis was applied. The percentage was generated for the number of respondents who selected each item on the questionnaire. Research Question 3 was answered by tallying the number of respondents that recorded each response. The mathematics coaches were asked on the questionnaire how they guided their students through the problem-solving process. They provided short phrases (some respondents provided more than one short phrase) as their response.

To answer Research Question 4, the mathematics coaches were asked what problem-solving strategies they taught to their students. The strategies that they listed were extracted as they were provided on the questionnaire by the respondents and placed into two categories: "known problem-solving strategies" and "unknown problem-solving strategies". The strategies that have been highlighted in the literature were placed in the category of "known problem-solving strategies" and the strategies that were not found in the literature were categorized as "unknown problem-solving strategies".

FINDINGS

Research Question 1: How do mathematics coaches describe a "problem-solving approach to teaching and learning of mathematics"?

Figures 1 to 3 represent the mathematics coaches' description of what it means to teach mathematics using a problem-solving approach. Each figure provides an example of the different perspectives the mathematics coaches have of teaching mathematics using a problem-solving approach. Of the 31 mathematics coaches, three of the respondents believed that teaching using a problem-solving approach represents a feature of novelty; 21 believed that teaching mathematics using a problem-solving approach is having students answer real life questions and seven of the respondents believed that having students engage with difficult tasks represents the use of a problem-solving approach to the teaching and learning of mathematics. An example that illustrates each of the three categories of responses was randomly selected and is represented in Figures 1 to 3.

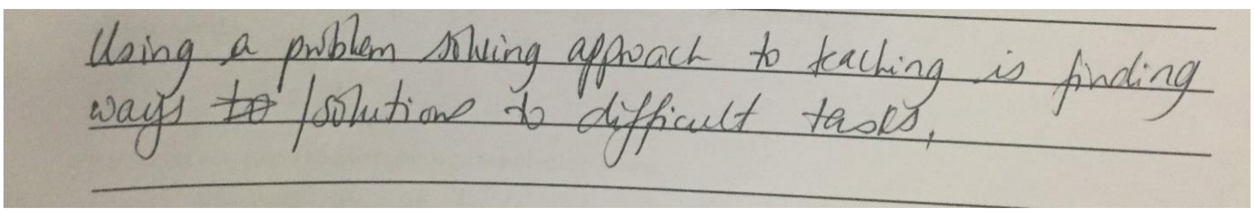
Using a problem solving approach to teaching is finding ways ~~to~~ /solutions to difficult tasks,

Figure 1.
Exemplar of a response to the question: Describe a problem-solving approach to teaching and learning of mathematics

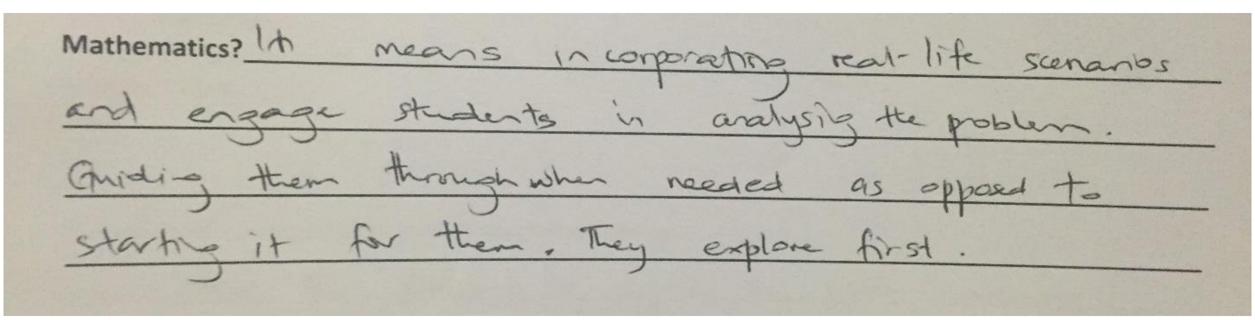
Mathematics? It means in corporating real-life scenarios and engage students in analysing the problem. Guiding them through when needed as opposed to starting it for them. They explore first.

Figure 2
Exemplar of a response to the question: Describe a problem-solving approach to teaching and learning of mathematics

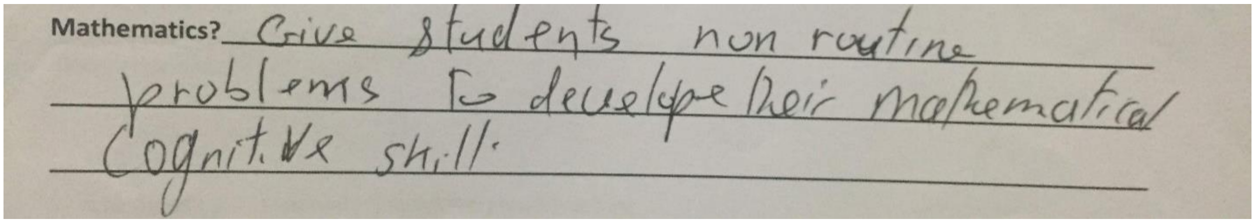
Mathematics? Give students non routine problems To develope their mathematical Cognitive skill.

Figure 3

Exemplar of a response to the question Describe a problem-solving approach to teaching and learning of mathematics

In Figure 1, the approach to teaching mathematics using a problem-solving approach means finding a solution to a difficult task; Figure 2 suggests that teaching mathematics using a problem-solving approach entails presenting students with real life scenarios, whilst Figure 3 suggests that teaching mathematics using a problem-solving approach entails presenting students with non-routine problems, in other words, problems that have some feature of novelty.

Research Question 2: What are mathematics coaches' perceptions of their preparedness to use a problem-solving approach to teach mathematics?

Table 1 shows the responses of the mathematics coaches.

It is evident from Table 1 (below) that the coaches perceived themselves as being well prepared to teach mathematics using a problem-solving approach. Over 90% of the respondents reported that they had the knowledge and skills required to teach mathematics using a problem-solving approach. That is, the respondents reported that they were aware of how to guide students through the problem-solving process; they were knowledgeable of the problem-solving strategies; they were aware of the teachers' role in facilitating problem-solving in the classroom; they knew how to choose appropriate problems/task/activities; and they knew how to implement problem-solving within lessons.

Table 1

Mathematics coaches' opinions on their preparedness for using a problem-solving approach in teaching mathematics

Criteria	# of coaches who selected agreed/strongly agreed	Percentage
Knowing how to use problem solving in teaching mathematics	30	97%
Possessing the required skills to teach using the problem-solving approach	29	94%
Knowing about the problem-solving process	30	97%
Knowing about problem-solving strategies	30	97%
Knowing how to organize problem-solving instructions	30	97%
Knowing one's own ability to implement problem-solving instructions	29	94%
Being aware of the teacher's roles in the problem-solving classroom	29	94%
Knowing how to choose appropriate problem-solving tasks/activities	28	90%
Being knowledgeable about where to source problem-solving task/activities	28	90%

Research Question 3: How do mathematics coaches guide students through the problem-solving process?

Of the 31 coaches, one responded that they use the *Socratic method*; another respondent stated that they allow students *to work on their own*; 18 responded that they *asked students questions*; one person responded that they *gave students clear instructions;* five reported that they *related the problem to students' experience* and, *had students read the problem several times*; six responded that they *identified what was given and what was required.*

It is noteworthy that the strategy recommended by Polya (1945) for teachers to use to guide students through the problem-solving process was never completely expressed by any of the mathematics coaches. For example, one mathematics coach expressed that he guided students through the problem-solving process by "asking students a series of questions". This represented only a fraction of one of the four heuristics recommended by Polya.

Research Question 4: What problem-solving strategies are taught by the mathematics coach?

Table 2 presents the problem-solving strategies that the mathematics coaches indicated that they used in their mathematics lessons

Table 2

Problem-solving strategies used by coaches in the mathematics classroom

Known problem-solving strategies	**Unknown problem-solving strategies**
Look for a pattern	Use of the 5 E's
Create a systematics list	Experiment
Trial and error	Re-read problem
Guesstimate	Brainstorming
Working backwards	Proofs
Deductive reasoning	Root cause analysis
	Hypotheses testing
	Read and evaluate
	Apply math concept
	Generate alternate solutions

It is notable that most of the problem-solving strategies that are listed in Table 2 are not discussed in the literature as problem-solving strategies.

DISCUSSION OF FINDINGS

Research Question 1: How do mathematics coaches describe a "problem-solving approach to teaching and learning mathematics"?

The findings show that the mathematics coaches' understanding and interpretation of what it means to use a problem-solving approach to teach mathematics differ. The findings are consistent with previous empirical research (e.g., Liljedahl et al., 2016; Lester & Kehle, 2003; Stein et al., 2003; Woodward et al., 2012).

The mathematics coaches' descriptions of their understanding of a problem-solving approach to teaching mathematics, for the most part, did not align with the features of non-routine, mathematics problems put forward by mathematics education researchers (e.g., Cai & Lester, 2010; Mayer, 2012; Schoenfeld, 1992; Woodward et al., 2012). Practitioners, inclusive of Polya, (1945), have all argued that a problem-solving approach to teaching mathematics is not merely having students' complete routine mathematics questions, but rather the problem must be novel, and the solution of the problem should not be one that is known or easily found (Cai & Lester, 2010; Chamberlin, 2008; Mayer, 2012; Schoenfeld, 1992). Some mathematics coaches believed that engaging students in difficult tasks suggests that they are engaged in problem solving. This provides evidence that, along with inconsistencies in the teachers' understanding of problem-solving, they also had an inaccurate understanding of what it means to use a problem-solving approach in the teaching and learning of mathematics.

Research Question 2: What are mathematics coaches' perception of their level of preparedness to use a problem-solving approach to teach mathematics?

The finding that 97% of the respondents perceived that they were prepared to teach mathematics using a problem-solving approach was promising. However, this finding is questionable since the mathematics coaches' responses regarding the strategies they used to enable students to develop problem-solving skills revealed some gaps in their knowledge of problem-solving pedagogy. The reasons for the contradiction between how the coaches perceived themselves and the shortcomings in their understanding of problem-solving pedagogy were not readily apparent.

Research Question 3: What problem-solving strategies are taught by the mathematics coaches?

Some of the problem-solving strategies for developing problem-solving skills in students that were listed by the mathematics coaches were never highlighted in the academic literature (unknown strategies). While it is understandable that teachers can use alternative methods to enable the development of problem-solving skills in learners, some of the strategies mentioned, such as use of the Five E's (which is a lesson plan model that is currently being enforced in the Jamaican context) are questionable as they do not reflect any of the critical features that would enable the development of problem-solving skills in learners. This gap in the teachers' understanding of the problem-solving strategies, to our knowledge, was not highlighted in the literature and so this finding is notable.

Research Question 4: How do mathematics coaches state that they guide students through the problem-solving process?

Similar to the problem-solving strategies, some of the methods the coaches stated that they used to guide students through the problem-solving process left questions as to whether teachers were truly prepared to teach mathematics using a problem-solving approach. For example, some teachers stated that they had guided students through the problem-solving process by "allowing students to work on their own". This does not align with early writing (e.g., Polya, 1945) or more recent literature (e.g. National Council of Teachers of Mathematics, 2014). In this regard, both of the aforementioned postulate that teachers of mathematics should guide students through the problem-solving process. This includes being able to take the appropriate action (or say the right thing) at the right time to get students to move forward through the problem-solving process. Yet, 97% of the teachers reported that they possessed the knowledge and skills required to guide students through the problem-solving process. Therefore, it cannot be taken for granted that teachers have the requisite knowledge and skills required to effectively facilitate students' development of problem-solving skills.

CONCLUSION AND RECOMMENDATIONS

The mathematics coaches' conceptualizations of problem solving, as well as how they guide students through the problem-solving process have provided key insights that might help to explain the shortcomings in students' performance in the Mathematics Problem-solving Competition. It also could potentially shed light on why national mathematics examinations, such as those administered by the Caribbean Examination Council, report that test takers display low levels of problem-solving skills.

The findings of this research imply that greater effort needs to be taken by key stakeholders within education, such as institutions tasked with preparing individuals to be mathematics teachers and the Ministry of Education which is responsible for driving and monitoring policy associated with how mathematics is taught. This is needed to ensure that teachers have a clear and common understanding of what it means to teach mathematics using a problem-solving approach, as well as how they can achieve success in guiding students through the problem-solving process.

Since problem solving is a key component of the STEM-related areas, immediate attention needs to be paid to teachers' inability to effectively facilitate students' development of problem-solving skills. In order for students to be able to perform satisfactorily in the STEM courses at the high school and university level, they need to be able to problem solve. In Jamaica and other countries, the dropout rate in the STEM related areas is high. Many university students are unable to manage the rigour of these areas because of their inability to think critically and problem solve.

Further research is needed in order to explain the contradictions that were observed in the findings. One case of this is that the mathematics coaches reported on the Likert scale that they perceived themselves as being prepared to teach mathematics using a problem-solving approach; yet some of the strategies which they reported using in the short answer section of the questionnaire were questionable (such as the use of five Es).

REFERENCES

Beghetto, R. A. (2017). Lesson unplanning: Toward transforming routine tasks into non-routine problems. *ZDM, 49* (7), 987-993.

Blömeke, S., Olsen, R. V., & Suhl, U. (2016). Relation of student achievement to the quality of their teachers and instructional quality. In T. Nilsen & J.-E. Gustafsson (Eds.), *Teacher quality, instructional quality and student outcomes: Relationships across countries, cohorts and time* (Vol. 2). International Association for the Evaluation of Educational Achievement.

Buschman, L. (2004). Teaching problem solving in mathematics. *Teaching Children Mathematics*, *10* (6), 302-309.

Cai, J., & Lester, F. K. (2010). *Why is teaching with problem solving important to student learning?* [Research brief]. National Council of Teachers of Mathematics.

Chamberlin, S. A. (2008). What is problem solving in the mathematics classroom? *Philosophy of Mathematics Education Journal*, *23* (1), 1-25.

Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches* (5th ed.). SAGE.

Darling-Hammond, L. (2010). *The flat world and education: How America's commitment to equity will determine our future*. Teachers College Press.

Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educational process*. Heath.

English, L., & Sriraman, B. (2010). Problem solving for the 21st century. In *Theories of mathematics education* (pp. 263-290). Springer.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics*, *66* (1), 61-75.

Hattie, J. (2003). *Teachers make a difference, what is the research evidence.* Paper presented at the Building Teacher Quality: What does the research tell us: Australian Council for Education Research Conference, Melbourne. http://research.acer.edu.au/research _conference_2003/4/

Hill, R. B. (1997). The design of an instrument to assess problem solving activities in technology education. *Journal of Technology Education, 9* (1), 31-46.

Jamaica Ministry of Education. (2013). National mathematics policy guidelines. http://bit.ly/2yi7PrI.

Leong, Y. H., Toh, T. L., Quek, K. S., Dindyal, J., Tay, E. G., & Tee, L. S. (2010). *Enacting a problem solving curriculum.* Paper presented at the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, Fremantle, Australia.

Lerman, S. (1983). Problem-solving or knowledge-centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology*, *14* (1), 59-66.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-804). Information Age Publishers.

Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, 10 (1/2), 245.

Lester, F. K., & Cai, J. (2015). Can mathematical problem solving be taught? Preliminary answers from thirty years of research. In P. Felmer, J. Kilpatrick, & E. Pehkonnen (Eds.), *Posing and solving mathematical problems: Advances and new perspectives* (pp. 117-135). Springer.

Lester, F. K., & Kehle, P. (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematical problem solving, learning, and teaching* (pp. 501-517). Erlbaum.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). Problem solving in mathematics education. In *Problem solving in mathematics education* (pp. 1-39): Springer.

Mapolelo, D. C., & Akinsola, M. K. (2015). Preparation of mathematics teachers: Lessons learnt from review of literature on teachers' knowledge, beliefs, and teacher education. *American Journal of Educational Research*, *3* (4), 505-513.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (2nd ed.). Pearson Education.

Mayer, R. (2012). Human non-adversary problem solving. In K. J. Gilhooly (Ed.), *Human and machine problem solving* (pp. 39-56). Plenum Press.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

Ngang, T. K., Nair, S., & Prachak, B. (2014). Developing instruments to measure thinking skills and problem solving skills among Malaysian primary school pupils. *Procedia-Social and Behavioral Sciences, 116* (2014), 3760-3764.

Perveen, K. (2010). Effect of the problem-solving approach on academic achievement of students in mathematics at the secondary level. *Contemporary Issues in Education Research, 3* (3), 9-14.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton University Press.

Posamentier, A. S., & Krulik, S. (2009). *Problem solving in mathematics, grades 3-6: Powerful strategies to deepen understanding*: Corwin.

Sakshaug, L. E., & Wohlhuter, K. A. (2010). Journey toward teaching mathematics through problem solving. *School Science and Mathematics*, *110* (8), 397-409.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Schoenfeld, A. H. (1987). Polya, problem solving, and education. *Mathematics Magazine, 60* (5), 283-291.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. Macmillan.

Schoenfeld, A. H. (2013). Reflections on problem solving theory and practice. *The Mathematics Enthusiast*, *10* (1/2), 9-34.

Stein, M. K., Boaler, J., & Silver, E. A. (2003). Teaching mathematics through problem solving: Research perspectives. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving grades 6-12* (pp. 245-256). National Council of Teachers of Mathematics.

Telese, J. A. (2012). Middle school mathematics teachers' professional development and student achievement. *The Journal of Educational Research, 105* (2), 102-111.

Toh, T. L., Seng, Q. K., Leong, Y. H., Dindyal, J., & Guan, T. E. (2011). *Making mathematics practical: An approach to problem solving.* World Scientific.

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., & Wray, J. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Pearson.

Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., . . . Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8. A practice guide*. National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. https://files.eric.ed.gov/fulltext/ED532215.pdf

Yazgan, Y. (2016). Fourth graders and non-routine problems: Are strategies decisive for success? *European Journal of Education Studies, 2* (4), 100-120.

Zollman, A. (2010). Commentary 2 on problem solving for the 21st century. In B. Sriraman and L. English (Eds.), *Theories of mathematics education* (pp. 297-301). Springer.

APPENDIX

Questionnaire: The Use of Problem-solving Approach to Teaching Mathematics

This questionnaire has 12 closed-ended and open-ended items. The scale on this questionnaire rates from 1 to 5, 1- strongly disagree; 2 – disagree; 3- undecided; 4 – agree; 5 – strongly agree. You are required to select one of the open-ended items.

Demographics:

Gender ________

Grade levels you teach ______________

Number of years teaching mathematics ______________

Qualification in mathematics _________________________ Other ____________________

Location of school (Parish) _____________________

Problem-solving Approach to Teaching

Describe what it means to you to use a problem-solving approach to teaching Mathematics.

		Agree	Strongly agree	Disagree	Strongly disagree	Undecided
1.	You introduce your mathematics lesson using a problem-solving approach	☐	☐	☐	☐	☐
2.	You use a problem-solving approach to deliver the mathematics content	☐	☐	☐	☐	☐
3.	You use a problem-solving approach to assess student mathematics learning	☐	☐	☐	☐	☐

4. How often would you say you use a problem-solving approach to teach mathematics?

____

Preparedness to teach mathematics using a problem-solving approach

		Agree	Strongly agree	Disagree	Strongly disagree	Undecided
1.	You have the required knowledge to teach mathematics using a problem-solving approach	☐	☐	☐	☐	☐
2.	You have the required skills to teach mathematics using a problem-solving approach	☐	☐	☐	☐	☐

3.	You are knowledgeable about the problem-solving process.	☐	☐	☐	☐	☐
4.	You are knowledgeable about the problem-solving strategies	☐	☐	☐	☐	☐
5.	You are knowledgeable about organizing problem-solving instructions.	☐	☐	☐	☐	☐
6.	You are knowledgeable about implementing problem-solving instructions	☐	☐	☐	☐	☐
7.	You are knowledgeable about choosing appropriate problem-solving tasks/activities.	☐	☐	☐	☐	☐
8.	You are knowledgeable about your role as a teacher during the problem-solving instructions.	☐	☐	☐	☐	☐
9.	You are knowledgeable about where to source appropriate problem-solving tasks/activities.	☐	☐	☐	☐	☐

10. How do you guide students through the problem-solving process?

10. What problem-solving strategies do you encourage your students to use?

Challenges with teaching using a problem-solving approach

12. What are the challenges you face with teaching mathematics using a problem-solving approach?

Authors

Avalloy McCarthy -Curvin (Mrs), Lecturer, UWI Mona (School of Education)

Lois George (PhD) Lecturer, UWI, Mona (School of Education)

Camella Buddo, (PhD) Lecturer, UWI, Mona (School of Education)

1. How do mathematics coaches describe a "problem-solving approach to teaching and learning mathematics"?
2. How do mathematics coaches guide students through the problem-solving process?
3. What are mathematics coaches' perceptions of their level of preparedness to use a problem-solving approach to teach mathematics?
4. What problem-solving strategies are taught by the mathematics coaches?